

Two-Photon-Exchange Effects and $\Delta(1232)$ Deformation

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The two-photon-exchange (TPE) contribution in $ep \rightarrow ep\pi^0$ with $W = M_\Delta$ and small Q^2 is calculated and its corrections to the ratios of electromagnetic transition form factors $R_{EM} = E_{1+}^{(3/2)}/M_{1+}^{(3/2)}$ and $R_{SM} = S_{1+}^{(3/2)}/M_{1+}^{(3/2)}$, are analysed. A simple hadronic model is used to estimate the TPE amplitude. Two phenomenological models, MAID2007 and SAID, are used to approximate the full $ep \rightarrow ep\pi^0$ cross sections which contain both the TPE and the one-photon-exchange (OPE) contributions. The genuine the OPE amplitude is then extracted from an integral equation by iteration. We find that the TPE contribution is not sensitive to whether MAID or SAID is used as input in the region with $Q^2 < 2 \text{ GeV}^2$. It gives small correction to R_{EM} while for R_{SM} , the correction is about -10% at small ϵ and about 1% at large ϵ for $Q^2 \approx 2.5 \text{ GeV}^2$. The large correction from TPE at small ϵ must be included in the analysis to get a reliable extraction of R_{SM} .

KEYWORDS: two-photon-exchange, pion production, $\Delta(1232)$ deformation

The study of the structure of the hadrons is one of the most important way to understand the non-perturbative properties of QCD. As the first excited state of the nucleon, the structure of $\Delta(1232)$ plays a special role as it has been established that Δ deforms. It provides a non-trivial test for theoretical models [1]. The electromagnetic excitation of the $\Delta(1232)$ provides a way to determine the deformation by measurement of the electric E2 and Coulomb C2 transitions in the electromagnetical (EM) production of pion in the $\Delta(1232)$ region. Recently, a few precise measurements of the multipoles M_{1+}, E_{1+} and S_{1+} related to this transition have been performed [2]. The precision of the measurements of the cross sections is close to 1% which implies that electromagnetic radiative corrections should be included in the theoretical analysis for the extraction to be reliable.

It has been established that the two-photon-exchange (TPE) effects contribute non-negligibly to the elastic ep scatterings [3] such that the TPE effects must be taken into account in the extraction of the ratio of the nucleon form factors G_E/G_M reliably. It is natural to ask how the TPE processes would contribute to the $\Delta(1232)$ excitation when aiming at precise determination of M_{1+}, E_{1+} and S_{1+} . The TPE corrections to the EM excitation of $\Delta(1232)$ at high Q^2 have been considered in [4] in partonic approach with the use of GPDs. However, TPE contributions at low Q^2 where the deformation of the $\Delta(1232)$ is inferred, have not been estimated. In addition, in [4], only the $\Delta(1232)$ pole contribution, but not the Born term, is considered for the one-photon-exchange (OPE) mechanism in calculating the interference between OPE and TPE amplitudes. However, it is well-known that the Born term plays important role within the OPE framework for pion EM production [1]. In this work, we estimate the TPE corrections to M_{1+}, E_{1+} and S_{1+} in the EM excitation of $\Delta(1232)$ at low momentum transfer Q^2 with the inclusion of the Born term in the OPE amplitude.

Within OPE approximation, the unpolarized cross section of the $ep \rightarrow ep\pi^0$ can be expressed as

$$\frac{d^5\sigma^{1\gamma}}{d\Omega_f dE_f d\Omega} = C |\mathcal{M}^{1\gamma} \mathcal{M}^{1\gamma*}| \equiv \Gamma \{ \sigma_0^{1\gamma} + \sqrt{2\epsilon(1+\epsilon)} \sigma_{LT}^{1\gamma} \cos \phi + \epsilon \sigma_{TT}^{1\gamma} \cos 2\phi \}, \quad (1)$$

where $\sigma_0^{1\gamma} = \sigma_T^{1\gamma} + \epsilon\sigma_L^{1\gamma}$, ϕ is the pion center of mass azimuthal angle with respect to the electron scattering plane, ϵ is the transverse polarization of the virtual photon, Γ is the virtual photon flux, and C is a kinematical constant. The virtual photon differential cross sections ($\sigma_T^{1\gamma}, \sigma_L^{1\gamma}, \sigma_{LT}^{1\gamma}, \sigma_{TT}^{1\gamma}$) are all functions of the center of mass energy W , four momentum transfer squared Q^2 , and the pion center of mass polar angle θ_π measured from the momentum transfer direction. When using the multipoles as inputs, the cross sections can be expressed as the functions of multipoles explicitly as $\sigma_{0,LT,TT}^{th}(Z_l, \theta_\pi)$, where Z_l 's denote the multipoles E_{l^\pm}, M_{l^\pm} and S_{l^\pm} . The superscript "th" is meant to emphasize that the expression $\sigma_{0,LT,TT}^{th}(Z_l, \theta_\pi)$ is fixed from theoretical consideration.

With the TPE contributions included, the unpolarized cross sections can be expressed as, in the same form as the OPE case of Eq. (1) [4],

$$\begin{aligned} \frac{d^5\sigma^{1\gamma+2\gamma}}{d\Omega_f dE_f d\Omega_\pi} &\simeq C\{|\mathcal{M}^{1\gamma}\mathcal{M}^{1\gamma*}| + 2\text{Re}[\mathcal{M}^{2\gamma}\mathcal{M}^{1\gamma*}]\} \\ &\equiv \Gamma\{\sigma_0^{1\gamma+2\gamma} + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}^{1\gamma+2\gamma}\cos\phi + \epsilon\sigma_{TT}^{1\gamma+2\gamma}\cos 2\phi\}, \end{aligned} \quad (2)$$

where the cross sections $\sigma_{T,L,LT,TT}^{1\gamma+2\gamma}$'s are now ϵ dependent. Within the OPE plus TPE approximation, $d\sigma^{1\gamma+2\gamma}/d\Omega_\pi$ would be what is measured experimentally and we may hence write

$$\frac{d\sigma^{ex}}{d\Omega_\pi} \simeq C\{|\mathcal{M}^{1\gamma}(X_{1^\pm}^{1\gamma}, \bar{Z}_{l^\pm}^{1\gamma})|^2 + 2\text{Re}[\mathcal{M}^{1\gamma*}(X_{1^\pm}^{1\gamma}, \bar{Z}_{l^\pm}^{1\gamma})\mathcal{M}^{2\gamma}]\}, \quad (3)$$

where $X_{1^\pm}^{1\gamma} = (E_{1^\pm}^{1\gamma}, M_{1^\pm}^{1\gamma}, S_{1^\pm}^{1\gamma})$ denotes the multipoles pertaining to the Δ excitation channel, while $\bar{Z}_{l^\pm}^{1\gamma}$ represent all other multipoles. The superscript 1γ means that they are the genuine multipoles extracted from the data with TPE effects removed. The functional dependence of $\mathcal{M}^{1\gamma}$ on multipoles (X_{1^\pm} and \bar{Z}_{l^\pm}) are well known.

The TPE diagrams we consider are depicted in Fig. 1. The crossed box diagram is not shown while the contact diagram of Fig. 1(b) is required by the gauge invariance [5]. As in [6], only the elastic intermediate states are taken into account in this study. Since we are interested in the TPE effects on the Δ peak $W = M_\Delta$, only the diagrams with Δ excited by the TPE are considered. In the evaluation of the diagrams of Fig. 1, vertices γ^*NN , $\gamma^*N\Delta$, $\gamma^*\gamma^*N\Delta$, and the Δ propagator are taken from [5] and [9]. The gradient coupling is used for $\pi^0 N\Delta$ vertex. The simple hadronic model as employed in [9] is followed to evaluate diagrams of Fig. 1 for the amplitude $\mathcal{M}^{2\gamma}$.

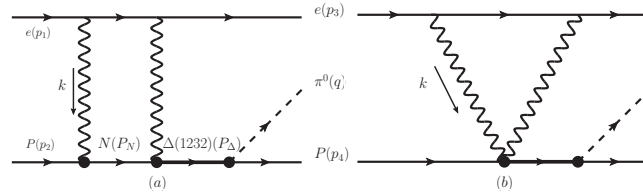


Fig. 1. TPE diagrams for $ep \rightarrow e\Delta \rightarrow ep\pi^0$, only the box and contact diagrams are showed, the x-box diagram is not showed.

To obtain the genuine OPE multipoles $Z_{l^\pm}^{1\gamma}$ from Eq. (3), we use the cross sections given by the two phenomenological models (MAID2007 [7] and SAID [8]) to approximate $\frac{d\sigma^{ex}}{d\Omega_\pi}$. In addition, $\bar{Z}_{l^\pm}^{1\gamma}$ is taken fixed such that only the three multipoles $X_{1^\pm} = (E_{1^\pm}, M_{1^\pm}, S_{1^\pm})$ remained to be determined. Furthermore, the phases of $E_{1^\pm}^{1\gamma}, M_{1^\pm}^{1\gamma}$ and $S_{1^\pm}^{1\gamma}$ are fixed to be the same as the respective input model as required by unitarity. This reduces the variables in Eq. (3) to only three numbers, namely, $|E_{1^\pm}|,$

$|M_{1+}|$, and $|S_{1+}|$. The nonlinear equation of Eq. (3) can be solved via iteration. The multipoles X_{1+} given by either MAID2007 (with $l \leq 8$) or SAID (with $l \leq 5$), depending on which model is used to approximate the experimental cross sections, are used for $M^{1\gamma}$ in Eq. (3), in the first iteration. We find that only one iteration is sufficient.

In the first iteration, we have the followings from Eq. (3),

$$\frac{d\sigma^{1,M/S}}{d\Omega_\pi} \equiv \left\{ \frac{d\sigma^{ex,M/S}}{d\Omega_\pi} - 2C \text{Re}[\mathcal{M}^{1\gamma}(X_{1\pm}^{0,M/S}, \bar{Z}_{l\pm}^{0,M/S}) \mathcal{M}^{2\gamma}] \right\} = C |\mathcal{M}^{1\gamma}(X_{1\pm}^{1,M/S}, \bar{Z}_{l\pm}^{1,M/S})|^2, \quad (4)$$

where superscript M/S refers to either MAID or SAID is used to approximate the cross sections and the corresponding multipoles. Superscripts 0 and 1 denotes the initial guess and the resulting multipoles obtained from first iteration, e.g., $X_{1\pm}^{0,M}, \bar{Z}_{l\pm}^{0,M} = X_{1\pm}(MAID), \bar{Z}_{l\pm}(MAID)$.

The problem now is how to determine the genuine OPE multipoles $X_{1\pm}^{1,M/S}$'s from Eq. (4) with TPE contributions removed. This can be done by first noting that the multipole dependence of $\sigma^{1,M/S}$ of Eq. (4) should be the same as $\sigma^{1\gamma}$ of Eq. (2), which we define as $\sigma_{0,LT,TT}^{th}(Z_l, \theta_\pi)$ earlier. We then determine the absolute values of $M_{1+}^{1\gamma}, E_{1+}^{1\gamma}$ and $S_{1+}^{1\gamma}$ by minimizing the following χ^2

$$\chi^2 \equiv \sum_{j=0,LT,TT} \sum_{\theta_i=1^\circ}^{179^\circ} \omega_j \{ \sigma_j^{1,M/S}(\theta_i) - \sigma_j^{th}(M_{1+}^{1,M/S}, E_{1+}^{1,M/S}, S_{1+}^{1,M/S}, \theta_i) \}^2 \quad (5)$$

where ω_j 's are the weights of cross sections $\sigma_{0,LT,TT}$ used in the fitting. We choose equal weights $\omega_j = 1$ in this preliminary study.

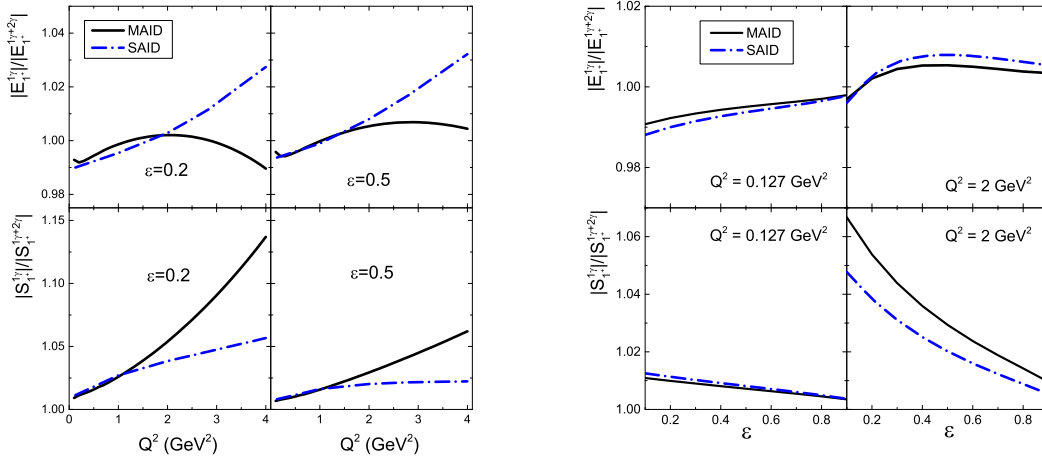


Fig. 2. TPE corrections to the extracted multipoles E_{1+} and S_{1+} with $\omega_j = 1$. The solid curves refer to the results with MAID2007 as input, and the dashed curves refer to the results with SAID as input.

We find that the TPE corrections to M_{1+} are very small. The TPE corrections to the multipoles E_{1+} and S_{1+} with the MAID2007 and SAID as inputs are presented in the Fig. 2, where the ratio between the multipoles $X_{1+}^{0,M/S}$ used in the input and the extracted genuine multipoles $X_{1+}^{1\gamma,M/S}$ after TPE effects are removed. Note that the multipoles $X_{1+}^{0,M/S}$'s which contain some TPE effects are labelled as $X_{1+}^{1\gamma+2\gamma}$ in the figures. It is seen that the ϵ dependence of the TPE corrections are similar with different models

as inputs, while the Q^2 dependence of the corrections are rather different for $Q^2 > 2 \text{ GeV}^2$. In this higher Q^2 region, the SAID multipoles agree better with the experiments.

The absolute TPE corrections to the R_{EM} and R_{SM} at fixed ϵ are presented in the Fig. 3 where $R_{EM} \equiv \text{Im}[E_{1+}]/\text{Im}[M_{1+}]$ and $R_{SM} \equiv \text{Im}[S_{1+}]/\text{Im}[M_{1+}]$. It is interesting to note that even though the obtained TPE corrections $S_{1+}^{1\gamma}/S_{1+}^{1\gamma+2\gamma}$ are very different with different input models in the $Q^2 \sim 2 - 4 \text{ GeV}^2$ region as seen in Fig. 2, the corrections δR_{SM} are almost same. When comparing our results with those given in [4], we see that when $\epsilon = 0.2$ our δR_{EM} are much smaller than theirs, while our δR_{SM} are much larger than theirs in the intermediate Q^2 . One of the main reason of this difference lies in the fact that only the Δ pole term is included for $M_{1\gamma}$ in [4]. Namely, the background contributions to $M_{1\gamma}$ are not considered there when calculating the interference term between $M^{1\gamma}$ and $M^{2\gamma}$.

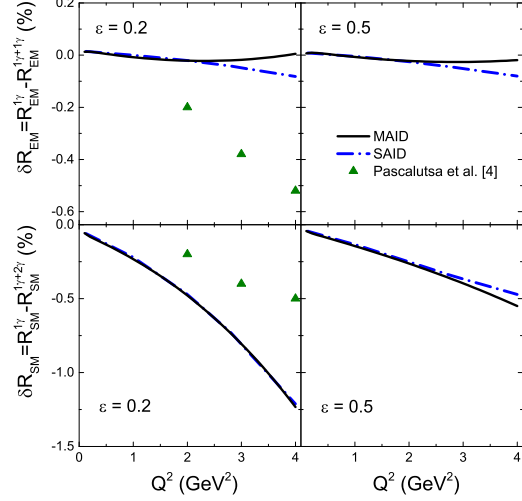


Fig. 3. TPE corrections $\delta R_{EM,SM} \equiv R_{EM,SM}^{1\gamma} - R_{EM,SM}^{1\gamma+2\gamma}$ with $\omega_j = 1$. Notations same as Fig. 2.

In summary, we evaluate the TPE corrections in the $ep \rightarrow ep\pi^0$ at $W = M_\Delta$ in the low Q^2 region in the hadronic approach. We include the background contribution in the OPE amplitude. We find that TPE corrections $\delta R_{SM} = R_{SM}^{1\gamma} - R_{SM}^{1\gamma+2\gamma}$ are not sensitive w.r.t. whether MAID2007 or SAID is used as input model. Our results differ considerably with [4] in $Q^2 \sim 2 - 4 \text{ GeV}^2$ region.

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